The Mesoscopic Physics of Diffusion MRI

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What does it mean to quantify microstructure?

Can we at least *distinguish* b/w 2 tissues?



Muscle fiber x-section



1 major direction

Similar diffusion tensor eigenvalues

Neuronal white matter fiber x-section

... at the very least, distinguish between these?



 $\Gamma \equiv 0, \ k < k_{\text{lat}}$

 $k\bar{a}/2\pi$

10

 10^{0}

10

10

Novikov, Fieremans, et al., PNAS 111, 5088 (2014)

Identical permeable barriers with same avg density \rightarrow same D_0 , D_{∞}









After coarse-graining: Slightly different D_i due to fluctuations in the # of barriers N_i

$$D_{
m inst}(t) - D_{\infty} \propto \langle (\delta D)^2 \rangle \propto \langle (\delta n)^2 \rangle \sim \frac{1}{\overline{N}} \sim \frac{1}{L(t)} \sim \frac{1}{\sqrt{t}}$$
 (Poissonian)
[Ernst et al, 1983]
 $\sim \frac{1}{t^{\vartheta}}, \quad \vartheta$ related to disorder class [DN et al., 2014]
 $\sim e^{-t/t_0}, \text{ periodic}$

How one really calculates this...

PNAS 111, 5088 (2014)

Coarse-grain down to $L(t) \rightarrow$ effective diffusion eqn $\partial_t \psi = \overline{D} \partial_{\mathbf{r}}^2 \psi + \partial_{\mathbf{r}} (\delta D(\mathbf{r}) \partial_{\mathbf{r}} \psi) \qquad D(\mathbf{r}) = \overline{D} + \delta D(\mathbf{r}), \quad \overline{D} \equiv \langle D(\mathbf{r}) \rangle$ $\frac{\mathbf{k}_{2}}{\mathbf{k}_{2}-\mathbf{k}_{1}} = G^{(0)}UG^{(0)} \to G^{(0)}_{\omega,\mathbf{k}_{2}} \mathbf{k}_{2} \cdot \mathbf{k}_{1} \delta D_{\mathbf{k}_{2}-\mathbf{k}_{1}} G^{(0)}_{\omega,\mathbf{k}_{1}}$ Disorder-averaged propagator up to $(\delta D)^2$ $G_{\omega,\mathbf{q}} = \int d\mathbf{r} \, dt \, e^{i\omega t - i\mathbf{q}\mathbf{r}} \, G_{t,\mathbf{r}} = \frac{1}{-i\omega + \overline{D}q^2 - \Sigma(\omega,\mathbf{q})} \qquad \qquad \Sigma(\omega,\mathbf{q}) \simeq \int \frac{d^d \mathbf{k}}{(2\pi)^d} \, \frac{[\mathbf{q}\cdot(\mathbf{k}+\mathbf{q})]^2 \Gamma_D(k)}{-i\omega + \overline{D}(\mathbf{k}+\mathbf{q})^2}.$ С Expand self-energy part up to q^2 ; obtain effective $D(\omega)$ as a pole of the propagator $\Gamma_D(\mathbf{r}) = \langle \delta D(\mathbf{r}_0 + \mathbf{r}) \delta D(\mathbf{r}_0) \rangle$ $G_{\omega,\mathbf{q}} = \frac{1}{-i\omega + D_{\infty}a^2 - \delta\Sigma(\omega,\mathbf{q})},$ $\Gamma_D(k) = \int d^d \mathbf{r} \ e^{-i\mathbf{k}\mathbf{r}} \ \Gamma_D(r)$ $\delta\Sigma(\omega,\mathbf{q}) = \frac{i\omega q^2}{D_{\infty}d} \int \frac{\mathrm{d}^d \mathbf{k}}{(2\pi)^d} \frac{\Gamma_D(k)}{-i\omega + D_{\infty}k^2} + \mathcal{O}(q^4). \qquad \frac{\mathcal{D}(\omega) - D_{\infty}}{D_{\infty}} \simeq -\frac{i\omega}{D_{\infty}^2d} \int \frac{\mathrm{d}^d \mathbf{k}}{(2\pi)^d} \frac{\Gamma_D(k)}{-i\omega + D_{\infty}k^2}$ In the time domain, $D_{\text{inst}}(t) - D_{\infty} \simeq \frac{1}{dD_{\infty}} \int \frac{d^d \mathbf{k}}{(2\pi)^d} \Gamma_D(k) e^{-D_{\infty}k^2 t} \sim t^{-\vartheta}, \quad \vartheta = \frac{p+d}{2}$ From correlator of coarse-grained $D(\mathbf{r})$ to correlator of structure $n(\mathbf{r})$: Simple proportionality! $\delta D(\mathbf{r}) \simeq (\partial D_{\infty} / \partial \overline{n}) \delta n(\mathbf{r})$

2

 t/τ

Diffusion = coarse-graining = low-pass filter

$$p = \infty$$

$$p = 2$$

$$p = 0$$

$$p = -(2 - \mu)$$

F.T.(barrier density correlation function) = power spectrum $\Gamma(k) = \int dx \, \langle n(x_0 + x)n(x_0) \rangle e^{-ikx} \propto |n(k)|^2$ $\Gamma(k) \sim k^p, \quad k \to 0$



Diffusion = coarse-graining over increasing L(t)~ Gaussian filter, selecting $k \leq 1/L(t)$

 $\log t \rightarrow \operatorname{small} k \rightarrow$ focus on universal features of structural organization

Novikov, Fieremans, et al., PNAS 111, 5088 (2014)

Identical permeable barriers with same avg density \rightarrow same D_0 , D_{∞}

 (D_{inst})

10





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|d = 1

Papaioannou, Novikov, Fieremans, Boutis Preprint <u>http://arxiv.org/abs/1607.08639</u> CUNY/NYU



Structural universality classes



Universality and dynamical exponents

The idea comes from theory of critical phenomena: ٠

At the phase transition, many systems behave in a similar way, with same **critical exponents**; Group them into "universality classes" L Landau, ~1930s; and A Migdal, V Pokrovsky, L Kadanoff, K Wilson, ~1970

Critical dynamics near a phase transition – **dynamical exponents** P Hohenberg, B Halperin, ~1980

In our case, the classification in terms of structural universality classes, • distinguished by the Brownian dynamics

$$D_{\rm inst}(t) = \frac{\partial}{\partial t} \frac{\langle x^2 \rangle}{2} \simeq D_{\infty} + {\rm const} \cdot t^{-\vartheta}$$



Structural exponent *p*:

p=0: uncorrelated (~Poissonian fluctuations) p>0: more ordered (smaller fluctuations); p<0: less ordered (larger fluctuations)

Finite $D_{_\infty}$ ightarrow Normal (not anomalous) diffusion, $\langle x^2
angle \sim t^1$, $t
ightarrow\infty$ • $\langle x^2 \rangle \sim D_{\infty} t + \text{const} \cdot t^{1-\vartheta < 1}$

The Real World...

New York University School of Medicine Department of Radiology

> 40+ MRI scanners ~ 200 MDs > 1.5•10⁶ exams/year

Center for Biomedical Imaging (660 First ave @38th st) = research division 100+ PhD researchers: RF coil development; fast acquisition methods (compressed sensing, MR fingerprinting); MR spectroscopy; tissue electrical properties mapping; mesoscopic MRI (diffusion, R₂*)



Radiology / MRI: What comes to mind?



Mesoscopic physics @ µm scale



mesoscopic Bloch-Torrey equation

arXiv:1612.02059 (2016)

$$\partial_t M = \partial_{\mathbf{r}} \left(D(\mathbf{r}) \partial_{\mathbf{r}} M \right) - i \Omega(\mathbf{r}) M - R_2(\mathbf{r}) M$$

Diffusion term: HC Torrey, PhysRev 1956 (e.g. cell walls, tumor vs benign tissue, etc)

Locally varying Larmor freq. offset (e.g. blood, iron)

Locally varying transverse relaxation rate (e.g. myelin vs water vs cytoplasm)

"MRI" & "microstructure" in PubMed



DN, VK, S.Jespersen (to appear)

Current state of dMRI in clinic

Early detection of regional cerebral ischemia in cats: comparison of **diffusion**-and T2-weighted **MRI** and spectroscopy

ME Moseley, Y Cohen, J Mintorovitch... - Magnetic ..., 1990 - Wiley Online Library Abstract Diffusion-weighted MR images were compared with T2-weighted MR images and correlated with 1 H spin-echo and 31 P MR Spectroscopy for 6-8 h following a unilateral middle cerebral and bilateral carotid artery occlusion in eight cats. Diffusion-weighted ... Cited by 1423 Related articles All 4 versions Web of Science: 1144 Cite More



 e^{-bD}

 $D\downarrow$

Clinical applications: Empirical so far

- Disease processes associated with diffusion abnormality:
 Ischemia, Demyelinating lesions, Tumors, Seizures, Drug toxicity, Abscess, Marrow abnormalities, Prion disease, ...
- Biophysical mechanisms underlying diffusion changes are often unclear; sensitive but not yet specific!
- E.g., the cause of the drop in the diffusion observed in acute stroke is still under debate: cell swelling, exchange, active transport mechanisms, axonal beading, etc.

Benveniste *et al*, Stroke 23 746 (1992); Nevo *et al*, NMR Biomed 23 734 (2010); Ackerman *et al*, NMR Biomed 23 725 (2010); Budde MD *et al* PNAS 107 14472 (2010); Fieremans E et al. ISMRM (2012), 3600; Hui ES et al. Stroke. 43 2968 (2012); Novikov DS et al, PNAS 111, 5088 (2014) **dMRI Goal:** "to see the invisible" To become <u>specific</u> to pathological changes at the mesoscopic scale, ~ $1 - 50 \mu m$

- Brain: Demyelination, axonal loss, inflammation...
 AD, MS, TBI, stroke
- Body: Cell size, membrane permeability, cell density...
 - Tumor grading (e.g. prostate)
 - Atrophy, rehabilitation, dystrophy (muscle)

Can we at least *distinguish* b/w 2 tissues?



Muscle fiber x-section



1 major direction

Similar diffusion tensor eigenvalues

Neuronal white matter fiber x-section

Focus on large-scale fluctuations

- short scale features gradually lost
- long range correlations survive



Muscle fiber x-section

Neuronal white matter fiber tract x-section

So, can we distinguish b/w these 2 tissues?



Novikov Nature Physics 2011 Novikov PNAS 2014

Cortical gray matter (rat)

Oscillating Gradient Measurements of Water Diffusion in Normal and Globally Ischemic Rat Brain

Mark D. Does,^{1*} Edward C. Parsons,² and John C. Gore^{1–3}







White matter tracts (human)







ain WM

ikov, et al. NeuroImage 2016

 D_{\parallel} D_{\perp}



5 healthy volunteers, 25-41 years old blos por portise of a initiation of the standard mai why words a first standard mai why words a first standard brai why words a first standard brai with the standard of the standard of the standard of the standard standard of the standard of





Human Skeletal Muscle (calf, shoulder)

Time Dependent Diffusion: Human Calf Muscle



nature physics

Random walks with barriers

Dmitry S. Novikov¹*, Els Fieremans¹, Jens H. Jensen^{1,2} and Joseph A. Helpern^{1,2,3†}









RPBM = Random permeable barrier model

3 Parameters: D_0 (w/o membranes) S/V surface-to-volume (to obtain size) κ membrane permeability

Scattering theory + real-space RG: *D*(*t*) for all *t*

Longitudinal Follow up 31 y/o M (Posterior Tibialis Tendon Tear)



Post-operative outcomes in rotator cuff muscle



- Setting of rotator cuff tendon
- Gold standard: muscular atrophy
- Outcomes diminish as rotator cuff atrophy and fatty infiltration worsen
- Current methods (CT, MRI) quantify shoulder atrophy <u>indirectly</u> as a function of fatty infiltration
- We focus on atrophy <u>directly</u>, via myofiber diameter using *D*(*t*)



D(t) in prostate cancer







Low Grade (3+3)



Intermediate Grade (3+4) High Grade (4+3)





Lemberskiy et al. Investigative Radiology 2017

Generalizations: $R_2^*(t) \sim \text{meso magnetic structure}$



Signature of jamming transition in dense bead suspensions

Ruh, Kiselev *et al*. Proc ISMRM 2015, p.1667



NIH R01 NS088040 Litwin Foundation for Alzheimer's Research Raymond & Beverly Sackler Fellowship

NYU

MRI Biophysics (est. 2012)

 ${}_{i\Omega(t,\mathbf{r})}M^{(t,\mathbf{r})}$

 $\partial_t M(t, \mathbf{r}) = \partial_r [D(\mathbf{r})\partial_r]$

diffusion-mri.com







alumni:

Conclusions

- Diffusion = coarse-graining
- Universality; power-law approach to D_{∞}
- In-vivo model validation tool

$$\vartheta = \frac{p+d}{2}$$



Applications:

- Stroke
- Neurodegeneration (MS, AD)
- Muscle atrophy
- Tumors



Ouantifying brain microstructure with diffusion MRI: Theory and parameter estimation

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We review, systematize and discuss models of diffusion in neuronal tissue, by putting them into an overarching physical context of coarse-graining over an increasing diffusion length scale. From this perspective, we view research on quantifying brain microstructure as occurring along the three major avenues. The first avenue focusses on the transient, or time-dependent, effects in diffusion. These effects signify the gradual coarsegraining of tissue structure, which occurs qualitatively differently in different brain tissue compartments. We show that studying the transient effects has the potential to quantify the relevant length scales for neuronal tissue, such as the packing correlation length for neuronal fibers, the degree of neuronal beading, and compartment sizes. The second avenue corresponds to the long-time limit, when the observed signal can be approximated as a sum of multiple non-exchanging anisotropic Gaussian components. Here the challenge lies in parameter estimation and in resolving its hidden degeneracies. The third avenue employs multiple diffusion encoding techniques, able to access information not contained in the conventional diffusion propagator. We conclude with our outlook on the future research directions which can open exciting possibilities for developing markers of pathology and development based on methods of studying mesoscopic transport in disordered systems.

		III. The $t \to \infty$ limit, regime (iii):	17
		Multiple Gaussian compartments	1/
	2	A. Neurites as sticks	18
an effectiv	e	1. Theory and assumptions	18
	2	2. Validation of the picture of sticks	18
ena	3	B. The Standard Model of diffusion in neuronal	
	3	tissue	19
or; qt		1. Theory	19
	5	Specificity and relevance of SM parameters	20
1		C. The challenge: SM parameter estimation	20
	8	1. SM parameter count	20
h scales?	8	2. How many parameters are necessary?	21
	9	A toy model of bi-modality: Parallel fibers	21
		Bi-modality beyond parallel fibers.	
	9	Flat directions in the fitting landscape	22
	10	D. SM parameter estimation using constrains	22
	10	1. White Matter Tract Integrity metrics (WMT)	i) 23
ie	11	2. Neurite Orientation Dispersion and Density	
To be or		Imaging (NODDI)	23
	12	E. ODF factorization and rotational invariants	24
	12	1. Isotropic $l = 0$ invariant $K_0(b)$	24
		2. Rotational invariants $K_l(b)$ for $l = 2, 4, \dots$	24
	13	F. Open questions: Precision and branch selection	25
	13	·····	
	13	IV. Multiple diffusion encodings	26
(ii):	15	A. MDE basics	26
ne (11):		B. Equivalence between MDE and SDE at $\mathcal{O}(q^2)$	27
	14	C Extra information relative to SDE at $\mathcal{O}(q^4)$ and	
	14	beyond Microscopic anisotropy	28
	14	D Concluding remarks on MDE	30
	10	D. Concluding femaliks on MDE	50
	17	V Outlook and open questions	30
		A To model or not to model?	30
		B. Ten problems for mesoscopic dMRI	31
		Acknowledgments	31
		References	32









[physics.bio-ph] CONTENTS I. Diffusion MRI through a bird's eye A. Mesoscopic Bloch-Torrey equation as a theory B. Coarse-graining and emergent phenome C. Diffusion as coarse-graining D. dMRI signal as the diffusion propagato Imaging E. Hierarchy of diffusion models based on coarse-graining: The three regimes F. How to become sensitive to short length G. Models versus representations H. The cumulant expansion as a default representation I. Normal or anomalous diffusion? II. Time-dependent diffusion in neuronal tissu A. Time dependent diffusion in the brain: not to be? B. Oscillating vs pulsed gradients C. The short-time limit, regime (i): Net surface area of restrictions 1. Theory 2. Applications D. Approaching the long time limit, regim Structural correlations via gradual coarse-graining 1. Theory 2. Applications E. Mesoscopic fluctuations * dima@alum.mit.edu [†] sune@cfin.au.dk

Review on dMRI modeling •

2016

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9

arXiv:1612.02059v1

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